Validity of the Quasi-Transparent Model of the Current Injected into Heavily Doped Emitters of Bipolar Devices

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***Abstract*—A simple criterion that permits one to assess the accuracy of the calculation of the current injected into a heavily doped emitter using the quasi-transparent model is presented. The criterion provides an upper limit of the error incurred by the approximation when compared to an exact computer solution, without requiring any additional calculations.**

The gain of a bipolar transistor is an important figure of merit that significantly constricts the design of optimized devices. In modern n-p-n silicon bipolar transistors, the maximum gain achievable is limited by the injection of minority carriers into the heavily doped n-type emitter. Analogously, high power conversion efficiency in solar cells can only be obtained through a minimization of all sources of recombination in the device, and among them, the recombination inside the heavily doped regions. The understanding of the physics of heavily doped emitters is essential to achieving the maximum potential from bipolar transistors and solar cells.

In both devices, the parameter that characterizes the recombination in the heavily doped emitter is the saturation current density J_0. While computer simulations of J_0 have been available for a long time, analytical solutions are very desirable since they provide valuable physical intuition. Until recently, the inhomogeneous doping distribution of practical heavily doped emitters had defied the construction of an analytical model. A perturbation approach, however, has been used by the authors of the present paper to elaborate a simple analytical model that was found to be very accurate for the shallow emitters of modern bipolar devices [1], [2].

The theory is based on the assumption that the dominant recombination mechanism in a heavily doped emitter occurs at the surface. In this assumption, the minority-carrier distribution inside the emitter is essentially fixed by their transport toward the recombining surface and is only weakly perturbed by the simultaneously occurring bulk recombination. The latter, therefore, can be easily calculated, to the first order, using the unperturbed minority-carrier profile. This approach leads to a very simple formulation that provides great insight into the complex physics of heavily doped emitters. The model was termed "quasi-transparent" because the minority-carrier distribution is calculated under the "transparent approximation" [3], i.e., total absence of bulk recombination. Due to its own nature, the quasi-transparent model becomes increasingly inaccurate as the magnitude of bulk recombination increases with respect to the surface recombination. This occurs as the overall doping level or emitter thickness increases. Higher order perturbation terms have recently been proposed to deal with this domain [4].

While the accuracy of any model can be assessed by comparison with an exact computer solution, the effort of developing the latter partially defeats the purpose of the elaboration of the analytical approximations. It is therefore very useful to establish the validity of the approximations without being forced to resort to the solution of the full problem. A simple criterion is presented in this brief.

The notation used in this paper is essentially identical to that of [1] and [2], but it has been made more transparent through the avoidance of the concept of "effective doping level" [5]. This rather unphysical entity is related to the physically meaningful equilibrium hole concentration p_0 through $p_0 = n_{i0}^2/N_{D\text{eff}}$, with n_{i0} being the intrinsic carrier concentration. In a practical 1-D n-type emitter, the hole current equation and the hole continuity equation (in the absence of generation) can be written, respectively, as [2], [5]

$$J_p = -qp_0 D_p \frac{du}{dx} \tag{1}$$

$$\frac{dJ_p}{dx} = -q\frac{p_0}{\tau_p} u \tag{2}$$

where

$$u = \frac{p - p_0}{p_0}. \tag{3}$$

The symbols have the same meaning as in [2].

Two boundary conditions to the problem exist. At $x = 0$ lies the space-charge-region edge of the injecting p-n junction. $x = W$ denotes the outside surface of the single-crystal semiconductor. The hole current at these two points can be expressed as

$$J_p(0) = J_0\left[\exp\frac{qV}{kT} - 1\right] = J_0 u(0) \tag{4}$$

$$J_p(W) = qS(p - p_0)|_{x=W} = qSp_0(W)\,u(W). \tag{5}$$

In (4), V is the forward voltage applied to the junction and J_0 is the emitter saturation current density, the proper figure of merit to this problem. In (5), S is the surface recombination velocity that characterizes the hole recombination rate at the outside surface.

The injected emitter current density is equal to the total recombination current inside the emitter, which can be obtained by integration of (2)

$$J_p(0) = J_p(W) + q\int_0^W \frac{p_0}{\tau_p} u\, dx. \tag{6}$$

The first term of the right-hand side represents the recombination current at the surface, with the second one being the recombining current in the bulk.

In the quasi-transparent approximation $J_p(W)$ and $u(x)$ are calculated in the absence of any significant bulk recombination. Through the use of (6), the emitter saturation current is obtained. In the simplified notation [5], it becomes

$$J_0 = \frac{q}{G_{\text{eff}}(W) + \dfrac{1}{Sp_0(W)}}\left\{1 + \int_0^W \frac{p_0}{\tau_p}[G_{\text{eff}}(W) - G_{\text{eff}}(x)]\, dx + \frac{1}{Sp_0(W)}\int_0^W \frac{p_0}{\tau_p}\, dx\right\} \tag{7}$$

where

$$G_{\text{eff}}(x) = \int_0^x \frac{dx}{p_0 D_p}. \tag{8}$$

This is the main result of the quasi-transparent model.

Note the two limits of (7). For high values of S, the surface-recombination velocity, one obtains

$$J_0(S = \infty) = \frac{q}{G_{\text{eff}}(W)}\left\{1 + \int_0^W \frac{p_0}{\tau_p}[G_{\text{eff}}(W) - G_{\text{eff}}(x)]\, dx\right\}. \tag{9}$$

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For low values of S, on the other hand,

$$J_0(S = 0) = q \int_0^W \frac{p_0}{\tau_p}\, dx. \tag{10}$$

The maximum error of the quasi-transparent model occurs precisely at this limit of negligible surface recombination rate where (10) applies. This lower limit violates the assumption of negligible bulk recombination with respect to surface recombination. However, as shown in [2], in the very practical case of shallow emitters, the model still predicts J_0 with high accuracy in this limit for the very practical case of shallow emitters. The reason lies in the fact that for shallow emitters with negligible surface recombination, the total recombination current is very small and the hole concentration throughout the emitter becomes large. The ratio J_p/p is then expected to be small, and therefore the hole quasi-Fermi level is essentially flat throughout the emitter. Equation (10) follows immediately [1].

The assumption of flat-hole quasi-Fermi level when $S = 0$ is equivalent to the zeroth order, or quasi-neutral quasi-equilibrium situation described in [4] and to assuming that $u(x)$ is constant throughout the emitter. From (1) one can estimate the quality of this approximation. Taking spatial averages in (1)

$$\left\langle \frac{du}{dx} \right\rangle = - \left\langle \frac{J_p}{q p_0 D_p} \right\rangle. \tag{11}$$

Approximately, the left-hand side is

$$\left\langle \frac{du}{dx} \right\rangle \simeq \frac{u(W) - u(0)}{W} \tag{12}$$

while the right-hand side is

$$\begin{aligned} \left\langle \frac{J_p}{q p_0 D_p} \right\rangle &= \frac{1}{W} \int_0^W \frac{J_p}{q p_0 D_p}\, dx \\ &< \frac{J_p(0)}{qW} \int_0^W \frac{dx}{p_0 D_p} < \frac{J_p(0)}{W J_0(S = \infty)} \end{aligned} \tag{13}$$

where we have used the fact that the hole current is maximum at the junction, and that $J_0(S = \infty)$ is an upper limit of $q/G_{\mathrm{eff}}(W)$ as (9) indicates.

Introducing (12) and (13) into (11), one obtains

$$u(W) - u(0) < \frac{J_p(0)}{J_0(S = \infty)}. \tag{14}$$

The relative error in the difference of the value of u from end to end of the emitter in the $S = 0$ case is, therefore, approximately

$$\frac{u(W) - u(0)}{u(0)} < \frac{J_p(0)}{u(0)\, J_0(S = \infty)} = \frac{J_0(S = 0)}{J_0(S = \infty)} \tag{15}$$

where the definition of J_0 given in (4) has been used.

The maximum error of the quasi-transparent model (which, as mentioned, appears for small values of S) is bracketed by the right-hand side of (15)

$$\epsilon < \frac{J_0(S = 0)}{J_0(S = \infty)}. \tag{16}$$

Both the numerator and denominator of (16) are calculated from the analytical model itself. They are, respectively, (10) and (9).

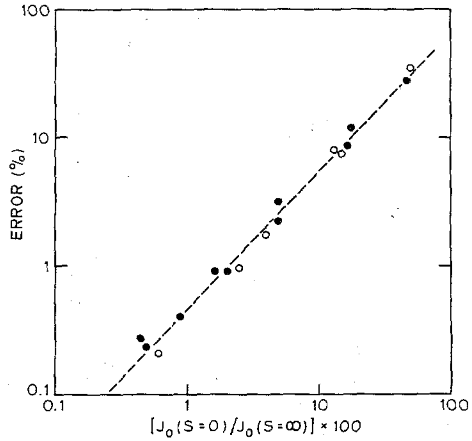

Fig. 1. Error of the calculation of J_0 in the analytical model with respect to an exact computer model, versus the ratio of J_0 at $S = 0$ and $S = \infty$ cm/s.

Therefore, there is no need to resort to an exact computer solution to estimate the accuracy of the model in any given application.

Indeed, Fig. 1 shows the comparison between the real error (obtained by simultaneously solving a given emitter by means of the analytical model and a computer solution that solves the transport equations [2], [5]) and the ratio (16). The comparison is carried out for a wide range of emitters, with different doping levels, profile distributions, and junction depths. Two different sets of physical models for the hole transport parameters have also been used. The closed circles correspond to calculations that used the models described in [2], while the open circles involve more accurate physical descriptions based on recent measurements [5], [6]. Irrespective of emitter shape, doping level, junction depth, or the values of the transport and recombination parameters, (16) always provides an upper limit for the error of the quasi-transparency approximation.

In conclusion, a simple validity criterion for the quasi-transparent model of the saturation current density of heavily doped emitters is proposed. The criterion provides "*a posteriori*" with an upper limit for the error incurred when using the analytical approximation, rather than an exact solution of the transport equations.

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