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Comments on "A Method for Determining Energy Gap Narrowing in Highly Doped Semiconductors"

JESUS A. DEL ALAMO AND RICHARD M. SWANSON

Abstract—**The narrow temperature range used by Neugroschel *et al.* [1] in their determination of bandgap shrinkage in heavily doped Si leads them to overestimate this parameter by neglecting its temperature dependence. Appropriate manipulation of their data results in values in better agreement with literature.**

In a recent paper [1] a method for determining bandgap shrinkage in heavily doped Si has been presented. The method is based on the measurement of the dc current injected into a heavily doped emitter at different temperatures. Previous reports of this work have appeared in the conference literature [2], [3].

The authors in [1] give the expression for the saturation current injected into a n-type homogeneouly doped transparent emitter

$$I_{QNE0} = A_E \frac{q n_i^2 D_p}{N_0(0)\, W_E} \frac{F_{1/2}(\eta_c)}{\exp(\eta_c)} \exp\left(\frac{\Delta E_G}{kT}\right). \tag{1}$$

In this formula, A_E is the emitter area, $N_0(0)$ is the surface impurity concentration, W_E is the emitter thickness, ΔE_G is the bandgap narrowing, and the term $F_{1/2}(\eta_c)/\exp(\eta_c)$ accounts for the necessity of using Fermi–Dirac statistics at the high doping levels considered. The remaining symbols have their usual meanings [1].

When the proper temperature dependence of n_i^2 is taken (around room temperature), (1) can be rewritten

$$C I_{QNE0}\, T^{-4} \frac{\exp(\eta_c)}{F_{1/2}(\eta_c)} = \exp\left[-\frac{E_{GI}(0) - \Delta E_G}{kT}\right] \tag{2}$$

where $E_{GI}(0)$ is the extrapolated zero-temperature bandgap [1], and the minority-carrier mobility has been assumed constant with temperature. Taking the logarithmic derivative in both sides with respect to $\beta = 1/kT$, they obtain

$$\frac{d}{d\beta} \ln \left\{ I_{QNE0}\, T^{-4} \frac{\exp(\eta_c)}{F_{1/2}(\eta_c)} \right\} = -\left[E_{GI}(0) - \Delta E_G\right] + \beta \frac{d\Delta E_G}{d\beta}. \tag{3}$$

Neugroschel *et al.* observe at this point that the term in brackets on the left-hand side of (3) plotted on a semilogarithmic scale gives a straight line whose slope is proportional to $E_{GI}(0) - \Delta E_G$, provided that ΔE_G is constant with temperature. Since in their experiments their data points follow a straight line, they conclude that indeed ΔE_G is constant with temperature in the temperature range investigated, and they extract the actual value of ΔE_G from the slope of this line.

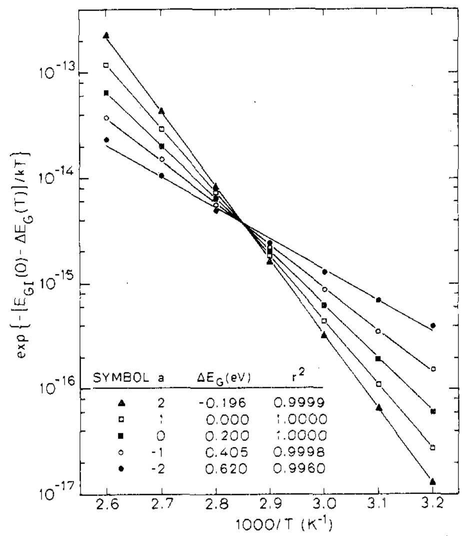

Fig. 1. Plot of $\exp\{-[E_{GI}(0) - \Delta E_G(T)]/kT\}$ versus $1000/T$ for $E_{GI}(0) = 1.206$ eV and $\Delta E_G(T) = 0.200\,(T/350)^a$ eV.

We show here that even if ΔE_G is strongly temperature dependent, the authors in [1] would never observe significant departures from linearity in the limited temperature range they consider. We plot in Fig. 1 the right-hand side of (2) (which differs from the quantity they plot only by a constant) for 7 points in $2.6 \leqslant 1000/T \leqslant 3.2$. An arbitrary temperature dependence of bandgap narrowing is taken: $\Delta E_G(T) = 0.200\,(T/350)^a$ eV, where we have selected typical values in their ΔE_G and T range.

We have then fitted these 7 points with an exponential line by a least-squares method. The resulting straight lines are also drawn in Fig. 1. The inset in Fig. 1 shows the extracted ΔE_G together with the regression coefficients of the least squares fittings. It is observed that, although for $-1 \leqslant a \leqslant 2$ the regression coefficients are excellent, the extracted values of ΔE_G are dramatically different. The sensitivity of the extracted ΔE_G to the temperature dependence of ΔE_G itself is so high that unless the latter is known with precision, the method presented in [1] will give unpredictable results.

In fact the values of ΔE_G obtained in [1] are considerably higher then those reported in the literature (even after the correction for degenerate statistics). We have graphically extrapolated I_{QNE0} at 300 K for devices UF2, UF12, and UF3 from Fig. 7 in [1]. Using (1) and the values of all the parameters therein, we have determined the minority-carrier diffu-

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TABLE I

SAMPLE	$N_o(0)$ (cm^{-3})	J_{QNEo} (300K) (Acm^{-2})	D_p^* (cm^2sec^{-1})	D_p[5] (cm^2sec^{-1})	ΔE_G (eV)	ΔE_G^{app} (eV)
UF2	$3x10^{19}$	$4.1x10^{-12}$	0.10	1.55	0.098	0.092
UF12	$1x10^{20}$	$1.6x10^{-12}$	0.21	1.45	0.138	0.118
UF3	$2.1x10^{20}$	$1.6x10^{-12}$	0.21	1.43	0.182	0.140

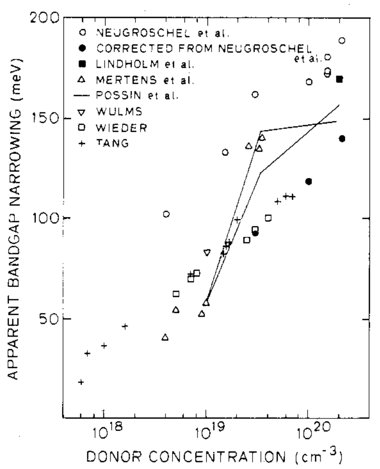

Fig. 2. Summary of measurements of ΔE_G^{app} versus impurity concentration for n-type Si. The open dots are the results reported in [1] adjusted with our (4). The solid dots correspond to diodes UF2, UF12, and UF3 corrected from [1] as indicated in the text.

sion coefficient at room temperature. The result is given in Table I under the heading D_p^*.

Mertens *et al.* [4] have obtained a diffusion coefficient for holes in $\simeq 3 \times 10^{19}$ cm^{-3} As-doped material which is around 9 to 11 times larger than the one calculated for sample UF2. No experimental results for D_p are available for more heavily doped silicon. However, the diffusion coefficient D_p for holes when they are majority carriers [5] is nearly one order of magnitude larger than D_p^*. This suggests to us that ΔE_G is overestimated, since if it were underestimated even smaller values of D_p^* would be required by (1).

A value of ΔE_G can be extracted for these samples by assuming the majority-carrier values [5] for the diffusion coefficients. Although this assumption is suspect in view of the actual state of knowledge of the band structure of heavily doped silicon [6], it provides valuable additional data in the high doping range in a way compatible with the literature. Using (1) again we obtain the results collected in Table I. To compare with the rest of measurements of bandgap narrowing we include the influence of degenerate statistics into an "apparent bandgap narrowing"

$$\Delta E_G^{app} = \Delta E_G - kT \ln \left[\frac{\exp(\eta_c)}{F_{1/2}(\eta_c)}\right] \tag{4}$$

which is plotted in Fig. 2, together with other measurements in the literature [4], [7]-[11] and the results in [1] adjusted using (4). The remarkable agreement with different reported data suggests that our interpretation is consistent with the common interpretation in the literature.

In conclusion, the narrow temperature range used in [1] lead Neugroschel *et al.* to conclude that within their range, the bandgap narrowing was temperature independent. Having shown here that in this limited temperature range this is not necessarily the case, we conclude that the amount of ΔE_G has been overestimated. Approximate single temperature values obtained from their data indeed give a lower ΔE_G in agreement with previous reported data in the literature.

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Reply[1] *by Arnost Neugroschel and Fredrik A. Lindholm*[2]

***Abstract*—The interpretation by del Alamo and Swanson of recombination-current data in highly doped silicon is clarified by making explicit their key assumptions. This leads to our reaffirming the merit of using temperature dependence in determining energy gap narrowing. This use gives newly determined low values of minority-carrier mobility and diffusivity, accompanied by a simple physical picture recently published elsewhere.**

In their comments [1] about our earlier work [2] on determining energy gap narrowing in highly doped Si, del Alamo and Swanson make two points. The first is a general point about activation energies which we believe is correct, as is discussed later. The second point involves their proposing that the values of energy gap narrowing in [2] are anomalous, accompanied by their reinterpretation of our data to remove the anomaly.

To recognize an anomaly, one needs a model. Our main purpose here is to suggest that del Alamo and Swanson rely on an

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